

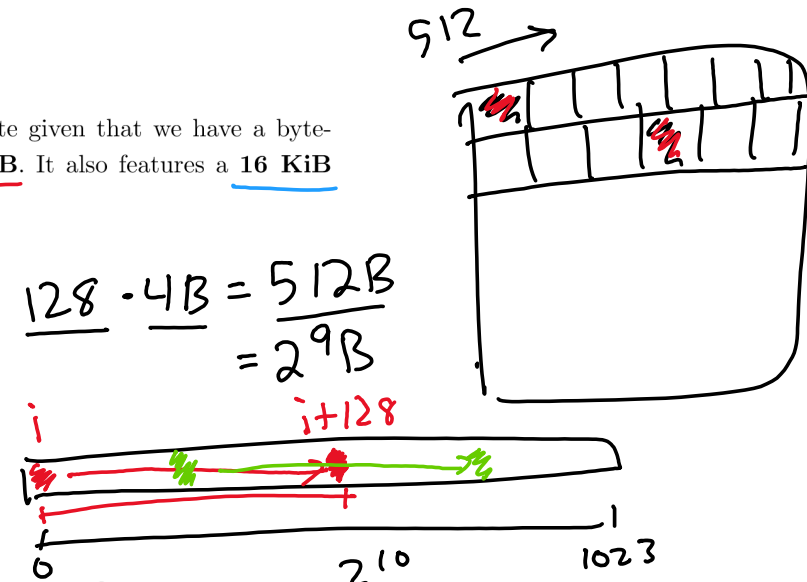
CS 61C  
Fall 2018

Caches II, Floating Point  
Discussion 6: October 1, 2018

1 Code Analysis

Given the follow chunk of code, analyze the hit rate given that we have a byte-addressed computer with a total memory of 1 MiB. It also features a 16 KiB Direct-Mapped cache with 1 KiB blocks.

```
#define NUM_INTS 8192 // 2^13
int A[NUM_INTS]; // A lives at 0x10000
int i, total = 0;
for (i = 0; i < NUM_INTS; i += 128) {
    A[i] = i; // Line 1
}
for (i = 0; i < NUM_INTS; i += 128) {
    total += A[i]; // Line 2
}
```



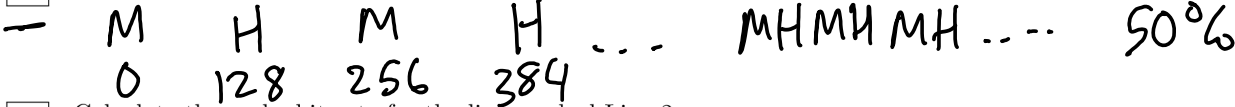
1.1 How many bits make up a memory address on this computer?

$1 \text{ MiB} = 2^{20} \text{ B}$        $\log_2 2^{20} = 20 \text{ bits}$

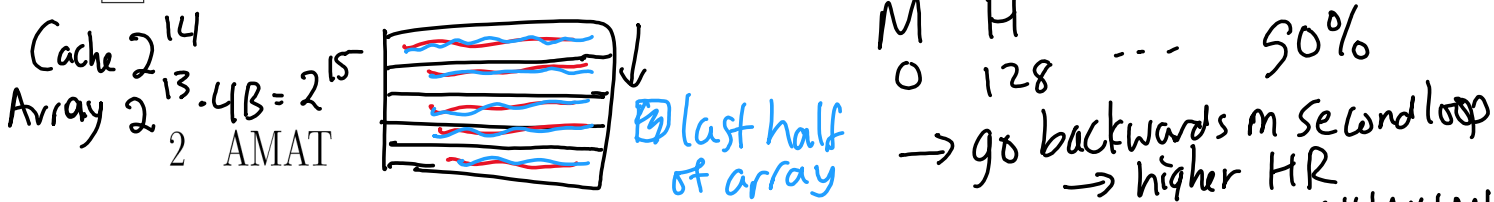
1.2 What is the T:I:O breakdown?

I:  $\frac{16 \text{ KiB}}{1 \text{ KiB}} = 16$        $\log_2 16 = 4 \text{ bits}$       O:  $1 \text{ KiB} = 2^{10}$        $\log_2 2^{10} = 10 \text{ bits}$

1.3 Calculate the cache hit rate for the line marked Line 1:



1.4 Calculate the cache hit rate for the line marked Line 2:

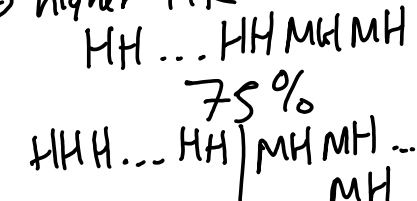


Recall that AMAT stands for Average Memory Access Time. The main formula for it is: check in cache - next cache or main Mem

$AMAT = \text{Hit Time} + \text{Miss Rate} * \text{Miss Penalty}$

We also have two types of miss rates, global and local. Global is calculated as: Fraction of ALL accesses that missed at that level over all accesses total. Whereas local is calculated: Fraction of ALL access that missed at that level over all access to that level total.

2.1 An L2\$, out of 100 total accesses to the cache system, missed 20 times. What is the global miss rate of L2\$?



$L = \frac{\text{Misses Here}}{\text{Accesses Here}}$

$G = \frac{\text{Misses Here}}{\text{Total Accesses}}$

$\frac{20}{100} = 20\%$

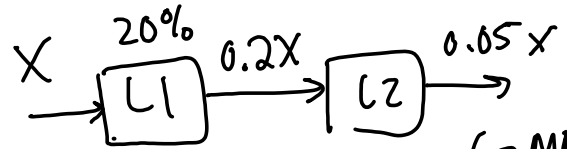
$$L1\ GMR = L2\ GMR$$

$$\frac{20}{50} = 40\%$$

2.2 If L1\$ had a miss rate of 50%, what is the local miss rate of L2\$?

Suppose your system consists of:

1. An L1\$ that hits in 2 cycles and has a local miss rate of 20%
2. An L2\$ that hits in 15 cycles and has a global miss rate of 5%
3. Main memory hits in 100 cycles



2.3 What is the local miss rate of L2\$?

$$L\ MR = \frac{.05X}{.2X} = .25 \quad 25\% = \frac{GMR}{L1\ GMR}$$

2.4 What is the AMAT of the system?

$$1 \cdot 2 + 0.2 \cdot 15 + 0.05 \cdot 100 = 2 + 3 + 5 = 10 \text{ cycles}$$

check L1                      check L2                      Access Mem Mem

2.5 Suppose we want to reduce the AMAT of the system to 8 cycles or lower by adding in a L3\$. If the L3\$ has a local miss rate of 30%, what is the largest hit time that the L3\$ can have?

### 3 Floating Point

The IEEE 754 standard defines a binary representation for floating point values using three fields:

- The *sign* determines the sign of the number (0 for positive, 1 for negative)
- The *exponent* is in **biased notation** with a bias of 127
- The *significand or mantissa* is akin to unsigned, but used to store a fraction instead of an integer

The below table shows the bit breakdown for the single precision (32-bit) representation.

↙ Bias: +127

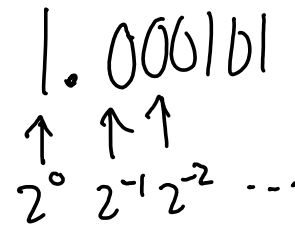
1	8	23
Sign	Exponent	Mantissa/Significand/Fraction

For normalized floats:

$$\text{Value} = (-1)^{\text{Sign}} * 2^{\text{Exp} - \text{Bias}} * 1.\text{significand}_2$$

For denormalized floats:

$$\text{Value} = (-1)^{\text{Sign}} * 2^{\text{Exp} - \text{Bias} + 1} * 0.\text{significand}_2$$



[-126, ]

Exponent	Significand	Meaning
0	Anything	Denorm
1-254	Anything	Normal
255	0	Infinity
255	Nonzero	NaN

← expo 000...0 0's for Mantissa  
 ← expo 111...1 ⇒ +/- ∞  
 ← expo 11...1 ⇒ NaN

3.1 How many zeroes can be represented using a float?

$$\begin{matrix} 00\dots0 & +0 \\ 10\dots0 & -0 \end{matrix} \Rightarrow \boxed{2}$$

$$254 - 127 = 127$$

$$1, \underbrace{11111111}_8 \times 2^?$$

3.2 What is the largest finite positive value that can be stored using a single precision float?

0 1111...1  $\Rightarrow$  NaN

0 1111 1110 1111...11  
 0x7F7FFFFF  $(2 - 2^{-23}) \cdot 2^{127}$

3.3 What is the smallest positive value that can be stored using a single precision float?

3.4 What is the smallest positive normalized value that can be stored using a single precision float?

3.5 Cover the following numbers from binary to decimal or from decimal to binary:

- 0x00000000
- 8.25 ①
- 0x00000F00
- 39.5625
- 0xFF94BEEF
- $-\infty$

① 1000.01  
 1.00001  $\cdot 2^3$   
 exp:  $3 + 127 = 130$   
 mantissa  
 sign 0000100...0

### 4 Extra Stuff on Caches!

4.1 Heres some practice involving a 2-way set associative cache. This time we have an 8-bit address space, 8 B blocks, and a cache size of 32 B. Classify each of the following accesses as a cache hit (H), cache miss (M) or cache miss with replacement (R). For any misses, list out which type of miss it is.

Address	T/I/O	Hit, Miss, Replace
0b0000 0100		
0b0000 0101		
0b0110 1000		
0b1100 1000		
0b0110 1000		
0b1101 1101		
0b0100 0101		
0b0000 0100		
0b1100 1000		

4.2 What is the hit rate of our above accesses?